

1-ST YEAR PLANS

CODE DEVELOPMENT. FULL GPU INTEGRATION TO THE CODES. LINEAR THEORY PREDICTIONS

1) Developing of a new GPU-based method for hybrid N-body/TVD approach (teams: UGR) Implementation of FFT GPU libraries for particle-mesh part of the code. Comparison of the new code with the parallel Tree-SPH code in terms of scalability, acceleration and accuracy. Design of the dynamic equilibrium of self-gravitating accretion disc model around supermassive black hole at time scales up to 10^8 yr.

2) Construction of equilibrium monotonic stellar models of spheroids within the black hole influence zone (team: R) Stability analysis requires self-consistent equilibrium models of stellar systems. Construction of such models is a separate problem unless the system geometry is spherically symmetric or razor flat.

3) Study of the possibility of gLCI in models in near-harmonic potentials away from the black hole (teams: UGR) According to theory, gLCI is possible when the orbit precession is retrograde, while the LC is a growing function of angular momentum. Polyachenko et al. (2010b) considered unstable DFs with polynomial dependence from angular momentum. With the use of numerical simulations, we plan to provide examples of realistic models subject to gLCI.
4) Study of methods for detection of unstable modes (teams: GR) Although different codes may provide similar final results of the evolution, some essential details can be missed. For example, relatively high level of Poisson noise typical for Tree-Code schemes can make it difficult to detect some stages of structure formation in stellar systems. Careful study of the most adequate numerical scheme for detection of unstable modes in disc and spherical systems, as well as techniques for its detection is needed.

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Non-self-consistent models

Black hole potential — spherical symmetry

 $\Phi(r) = -\frac{GM_{\rm C}}{r}$

DF — axial symmetry

$F_0(E, L_z, I_3) \Rightarrow F_0(E, L_z, L) \qquad L \approx \text{const}$

Potential produced by the cluster is neglected

$$\delta \Phi(r,\theta) \ll \Phi(r)$$

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Non-self-consistent models – Example 1 $F = A_n \delta(E - E_0) L_z^n$ n = 40.8 0.6 Volume density: 0.4 $\rho(r,\theta) = B_n \, \frac{M}{R^3} \, \times \,$ $\times \left(\frac{r}{R}\sin\theta\right)^n \left(\frac{R}{r}-1\right)^{(n+1)/2}$ -0.6 -0.8 -1-0.8 -08 -06 -0.4 -0.2 0 0.2 0.4 0.6

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<u>Non-self-consistent models – Example 2</u>

$$F = A_{nm} \delta(E - E_0) \, rac{L_z^n}{L^m}$$

Volume density:

$$\rho(r,\theta) = B_{nm} \frac{M}{R^3} \left(\frac{r}{R}\right)^{n-m} \left(\frac{R}{r}-1\right)^{(n-m+1)/2} \sin^n \theta$$

Location of the maximum:

$$r = R_{\max} = \frac{n - m - m}{2}$$

$$2(n-m)$$

Tn

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Self-consistent models

Axisymmetric potential

$$\Phi(r) = -\frac{GM_{\rm C}}{r} + \delta\Phi(r,\theta)$$

How DF is transformed? No problem for

$$F_0(E,L_z)$$

In case of the near-Keplerian potentials, the DF

$$F_0(E, L_{oldsymbol{z}}, I_3)$$

can be constructed by perturbation theory using canonical transformation

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Self-consistent models

Usual action-angle variables:

$$(I_1, I_2, I_3; W_1, W_2, W_3)$$

$$I_1 = I_r, \quad I_2 = L - |L_z|, \quad I_3 = L_z$$

Delaunay variables (B&T, Appendix E):

$$(J_1, J_2, J_3; w_1, w_2, w_3)$$

$$I_1 = J_1 - J_2, \quad I_2 = J_2 - |J_3|, \quad J_3 = I_3$$

Hamiltonian:

$$H = -\frac{(GM_{\rm C})^2}{2(I_1 + I_2 + |I_3|)^2} = -\frac{(GM_{\rm C})^2}{2(I_r + L^2)} = -\frac{(GM_{\rm C})^2}{2J_1^2}$$



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Self-consistent models

Generating function:

$$S(\mathbf{w}', \mathbf{J}) = \mathbf{w}' \cdot \mathbf{J} + s(\mathbf{w}', \mathbf{J}), \quad s(\mathbf{w}', \mathbf{J}) = \mathcal{O}(\epsilon)$$

Corrections:

$$E' = E + \delta \Phi(r, \theta), \quad L' \equiv J'_2 = L + \Delta L, \quad L'_z = L_z$$

$$\Delta L(E, L, L_z; r, \theta) = \Delta J_2 = \frac{4}{\Omega_1} \sum_{l=0}^{l_{\max}} T_l \sum_{l_1=1}^{\infty} \sum_{\substack{l_2 > 0, \\ l_2 \text{ even}}} (l_2/l_1) \times \left[\phi_{l_1 l_2}^{A} \cos(l_1 w_1) \cos(l_2 w_2) - \phi_{l_1 l_2}^{B} \sin(l_1 w_1) \sin(l_2 w_2) \right]$$

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Polynomial model

DF:

$$F = N\delta(E - E_0)\alpha^n$$
, $\alpha \equiv L/L_{circ}$

Volume density:

$$\rho = C_n x^n (1 - x^2)^{(n+1)/2}, \quad x \equiv r/R$$

Potential (near-h + self-g):

$$\Phi_0(r) = \frac{\Omega_0^2 r^2}{2} + \Phi_G(r)$$

Units:

$$G = M_G = R = 1 , \quad \Omega_0 = 10$$

Timescales:

$$t_1 \sim \Omega_0^{-1} = 0.1$$
, $t_2 \sim (\epsilon \Omega_0)^{-1} = 10$, $\epsilon \equiv \frac{GM_G}{R^3 \Omega_0^2}$

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Polynomial model - N-body experiments (1M)



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Radial equilibrium:

$$R\Omega^2(R) = \frac{d}{dR} \left[\Phi_0(R) + h_0(R) \right]$$

Linear perturbations:

$$-i\omega_* v_R - 2\Omega v_\theta = -\frac{a}{dR}(\Phi + h)$$
$$-i\omega_* v_\theta + \frac{\kappa^2}{2\Omega} v_R = -\frac{im}{R}(\Phi + h)$$

$$-i\omega_*\Sigma + \frac{1}{R}\frac{d}{dR}(R\Sigma_0 v_R) + \frac{im\Sigma_0}{R}v_\theta = 0$$

 $h = c_s^2 \frac{\Sigma}{\Sigma}$

 $\omega_* \equiv \omega - m\Omega$

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 $= -i\omega_*v_R$

Standard approach (no self-grav., finite boundaries):

 $\frac{d}{dr}(c_s^2\eta) = \frac{2m\Omega}{r\omega_*}(c_s^2\eta) - (\kappa^2 - \omega_*^2)\xi$

 $\frac{d}{dr}(rH_0\xi) = -rH_0\left[\left(1 - \frac{m^2c_s^2}{r^2\omega^2}\right)\eta + \frac{2m\Omega}{r\omega}\xi\right]$

Boundary conditions — rigid walls:









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Unknown frequency is in the l.h.s only:

$$\omega v_R = m\Omega v_R + 2i\Omega v_\theta - i\frac{d}{dR}\left(\hat{H}_m\Sigma\right)$$

$$\omega v_{\theta} = -i\frac{\kappa}{2\Omega}v_{R} + m\Omega v_{\theta} + \frac{m}{R}\hat{H}_{m}\Sigma \qquad \qquad \vdots \quad \omega \mathbf{M}\mathbf{x} = \mathbf{L}\mathbf{x}$$

$$\omega \Sigma = -\frac{1}{R} \frac{d}{dR} \left(R \Sigma_0 v_R \right) + \frac{m \Sigma_0}{R} v_\theta + m \Omega \Sigma \quad \mathbf{A} = \mathbf{M}^{-1} \mathbf{L}$$

GPU!

$$\hat{H}_m \Sigma \equiv (\hat{G}_m + c_s^2 / \Sigma_0) \Sigma = (\Phi + h)$$

What about boundary conditions?



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Example II — "Exactly solvable model" (Hunter 1963, Shukhman)

Surface density:

$$\Sigma_0(R) = \Sigma_* \xi , \quad \xi \equiv \left[1 - R^2 / a^2 \right]^{1/2}$$

Potential:

$$\Phi_0(R) = \frac{\Omega_D^2 R^2}{2} , \quad \Omega_D^2 = \frac{\pi^2 G \Sigma_s}{2a}$$

Pressure law:

$$p_0(R) = \frac{1}{3} \Sigma_* c_*^2 \xi^3$$

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Example II — eigenmodes

Eigenfunctions of surface density:

$$\Sigma_n(R) = \Sigma_* \frac{P_n^m(\xi)}{\xi} e^{im\theta}$$

Eigenvalues are obtained from the cubic DR:

$$=\frac{c_*^2/a^2-4\Gamma_n^m\Omega_{\rm D}^2}{\omega_*^2-4\Omega^2}\left(n^2+n-m^2-\frac{2m}{\omega_*}\right)$$

$$\Gamma_n^m = \frac{(n+m)!(n-m)!}{2^{2n+1} \left[\left(\frac{n+m}{2} \right)! \left(\frac{n-m}{2} \right)! \right]^2} , \quad n = 2, 4, 6, \dots$$

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Example II — Convergence on N and β to 0.5 + 0.61237244i

			$-\lg k$
N	10 ³ Δ ₃	10 ³ Δ ₄	10 ³ Δ ₅
4	8.7+29i	4.5+16i	1.7+2.4i
8	5.2+6.7i	4.4+14i	2.1+4.1i
16	2.4-13i	4.6+8.7i	2.8+4.2i
32	0.84-17i	2.7+4.0i	2.5+5.1i
64	0.68-15i	0.26-1.6i	1.4+3.8i
128	0.43-16i	0.008-2.2i	0.41+1.2i
256	0.36-16i	0.012-2.1i	-0.06-0.4i
512	0.32-16i	-0.016-2.3i	-0.03-0.33i
1024	0.26-16i	-0.08-2.5i	-0.09-0.58i

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Example III — cored exponential model (galactic disc)

Potential:

$$\Phi_0(R) = v_0^2 \ln \sqrt{1 + R^2 / R_C^2}$$

Surface density:

$$\Sigma_D(R) = \Sigma_s \exp\left[-\lambda \sqrt{1 + R^2/R_C^2}\right], \quad \lambda \equiv \frac{R_C}{R_T}$$

T

The stellar dynamical model F₀(E,L) has 3 parameters

$$\sigma_R pprox v_0 / (2\overline{N})^{1/2}$$

Units and adopted parameters:

$$G = v_0 = R_C = 1$$
 $\overline{N} = 6, \ \lambda = 0.625, \ \Sigma_s = 0.34$

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Example III — propagation diagrams (enhanced WKB, Bertin+89)





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Swing amplified outer modes? Presumably yes

When R_{out} is increased from $10R_d$ to $20R_d$;

- frequency range corresponds to the max of SA growth;
- travel time for wave packets increased by factor 2.33;
- growth rates dropped by factor 2.28;
- ΔΩ dropped by factor 2.31

$$n\lambda = 2(R_{
m out} - R_{
m C})$$

1-ST YEAR SUMMARY

- 1) (#2 "Spheroids, I₃") theoretical part, numerical part in progress (exp.: fall 2017)
- 2) (#3 "Loss cone inst.") negative results. Publication?
- 3) (#4 "Mode's detection") used everywhere no separate publication.
- 4) Papers with acknowledgements to the project:
 - Polyachenko E., Berczik P., Just A. On the bar formation mechanism in galaxies with cuspy bulges // MNRAS, vol. 462, Issue 4, p.3727-3738 // arXiv:1601.06115
 - Polyachenko E., Berczik P., Just A. Bar formation in the Milky Way type galaxies // Baltic Astronomy, vol. 25, No. 4, pp 411-418 // arXiv:1702.01646
 - Polyachenko E. Swing amplification and global modes reciprocity in models with cusps // Baltic Astronomy, vol. 25, No. 3, pp 288-295 // arXiv:1608.01776
 - Polyachenko E.V., Shukhman I.G. Radial orbit instability in systems of highly eccentric orbits: Antonov problem reviewed // MNRAS, accepted May 24 (2017) // arXiv:1705.09150
 - Polyachenko E.V., Shukhman I.G. Instability of stationary spherical models with orbits arbitrarily close to radial // Baltic Astronomy, vol. 25, No. 4, p. 362-368
 - Polyachenko E.V. The linear eigenvalue problem for barotropic selfgravitating discs // MNRAS, submitted May 08 (2017) // arXiv:170X.XXXXX

2-ND YEAR PLANS N-BODY/HYDRODYNAMIC SIMULATIONS

1) Self-consistent N-body/hydrodynamical simulations of the AD imbedded in the nuclear stellar cluster (teams: UGR) Implementation of the AD heating due to interaction of the stellar population of NSC. Calculation of the accretion rate on to the BH due to dynamical friction of stars with the AD matter. Comparison of the results with the previous studies and linear theory predictions (Just et al 2012). Calibration of the BH accretion rate with various subgrid models (Springel et al. 2005; Okamoto et al. 2008; Debuhr et al. 2011).

2) Theoretical analysis of the possible angular momentum distributions of the stellar clusters (teams: GR) Dependence of the stellar distribution function (DF) on the angular momentum is essential for stability. In early numerical experiments only monotonic DFs were studied (e.g., Cohn, Kulsrud 1978). However, we expect that non-monotonic distributions are also feasible. If a cluster is formed as a result of the collisionless collapse, then it remains collisionless until the collisional relaxation will take over (see, e.g., Merritt & Wang, 2005). Thus in principle, the system can have almost arbitrary DF both in the energy and in the angular momentum. We wish to analyze the variety of possible distributions over angular momentum in evolutionary models.
3) Numerical simulations of the possible angular momentum distributions of the stellar clusters (teams: UG) The same as in task 2, but using numerical simulations.

4) Study of the possibility of the loss cone instability for spherical systems in near-Keplerian potentials (teams: UGR) Theory gives constrains on the possible unstable spherical harmonics and width of the loss cone. Using tasks 2 and 3, we shall try to present reasonable stationary initial conditions that promote gLCI.