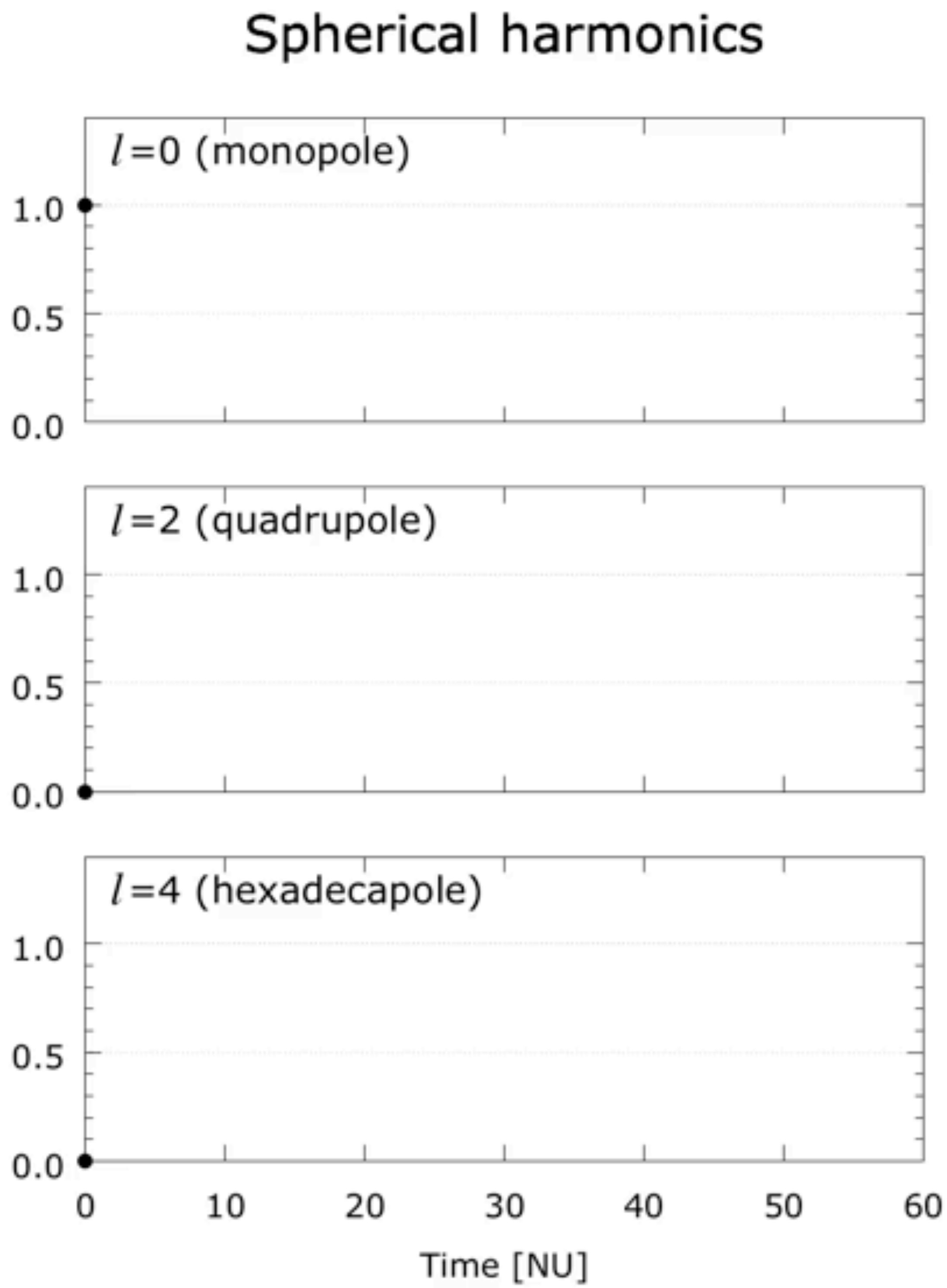
Radial-Orbit And
LOss Cone Instabilities
In Spherical Systems

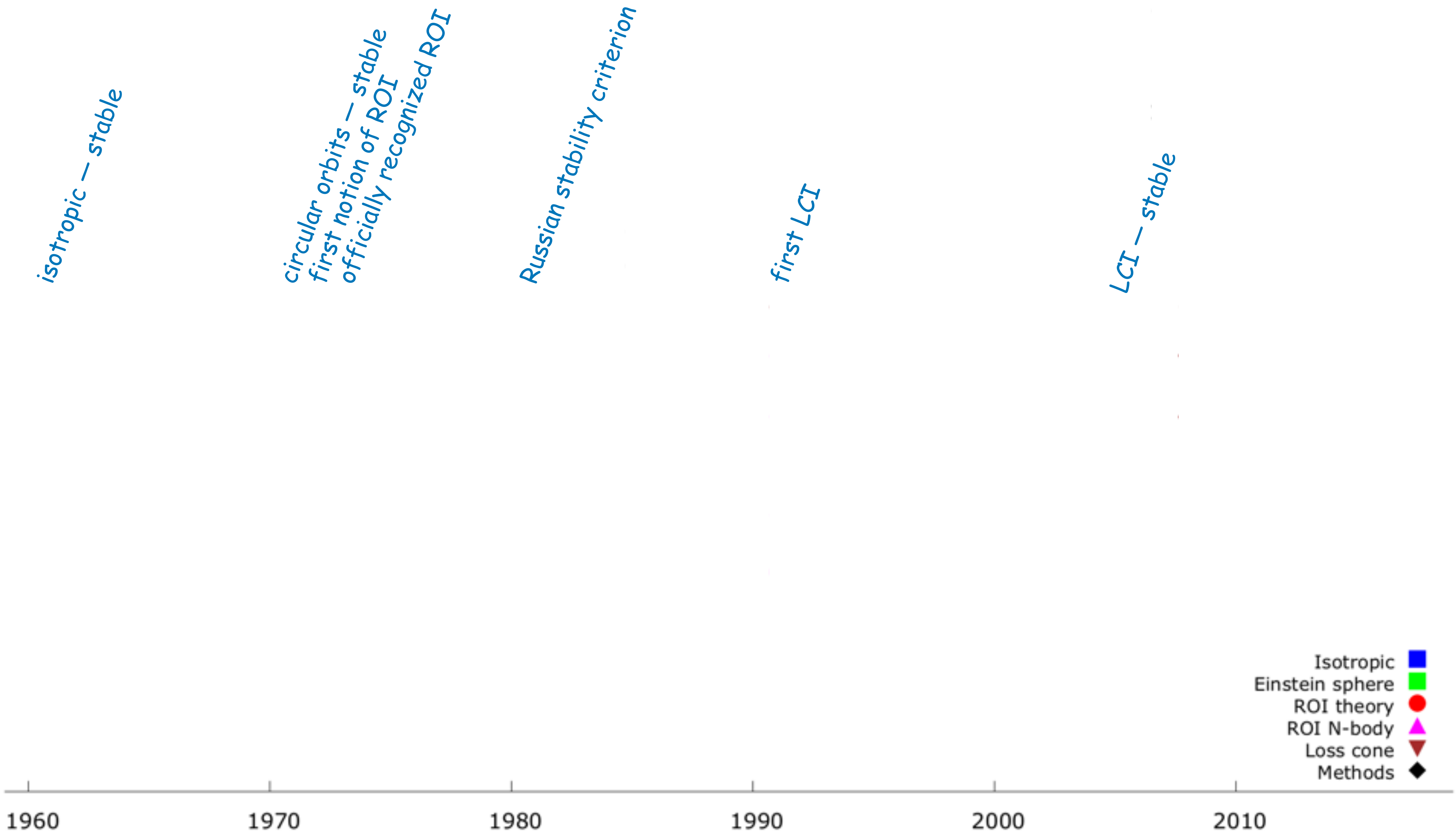
Evgeny Polyachenko
Institute of Astronomy RAS, Moscow, Russia

Example of an instability



P & Berczik

Stellar spheres — timeline



Topics

- Radial-orbit instability (ROI) — properties;
- ROI — inappropriate title;
- Loss cone instability (LCI) — properties + N-body

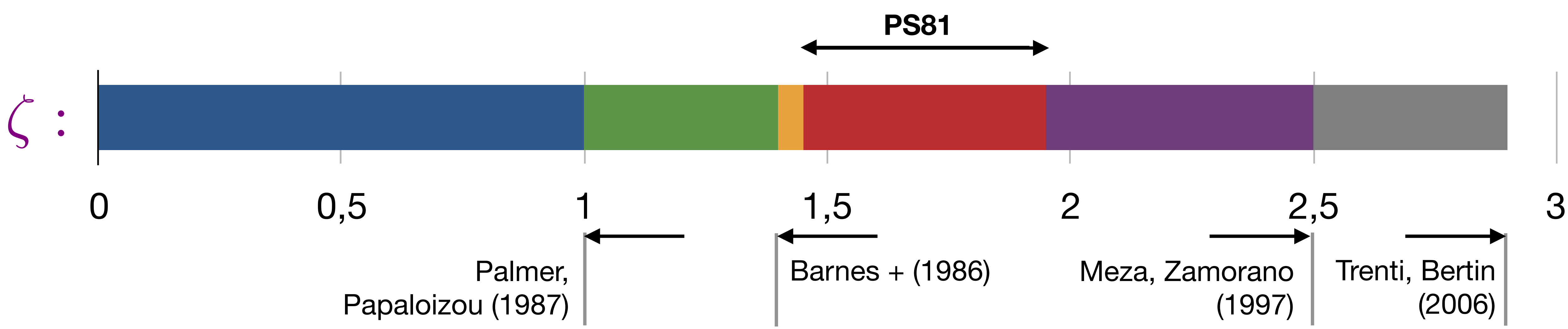
Russian stability criterion

- Matrix method for spheres (á la Kalnajs for discs, **nonlinear** w.r.t. **ω**)

- Stability criterion (á la Ostriker—Peebles)

ratio of the total kinetic energy of radial and transverse motion

$$\zeta \equiv \frac{2T_r}{T_\perp} \qquad \zeta_{\rm crit} = 1.7 \pm 0.25$$



Polyachenko & Shukhman (1981); Polyachenko (1983)

Mechanism — Jeans or Lynden-Bell?

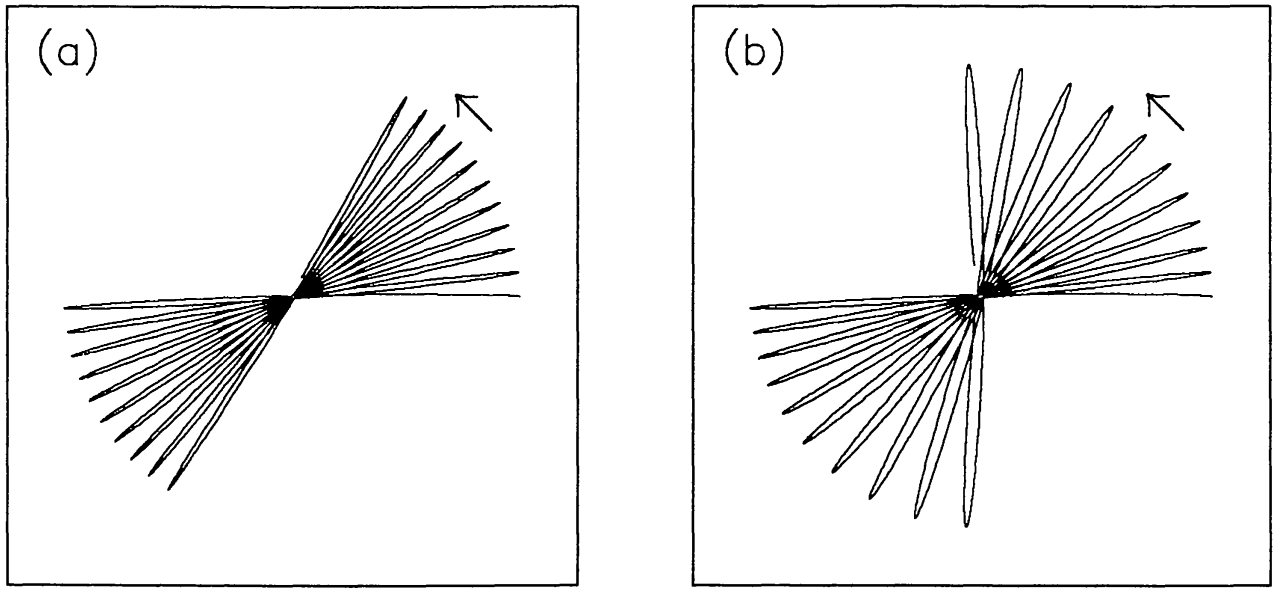
- Jeans instability in the anisotropic media

  Antonov (1973), Barnes et al. (1986), P. & Shukhman (2015, 2017)

- Orbital paradigm (similar to bar formation in discs, Lynden-Bell 1979)

  Merritt(1987), Palmer & Papaloizou (1987), Saha (1991), Weinberg (1991), Palmer (1994)

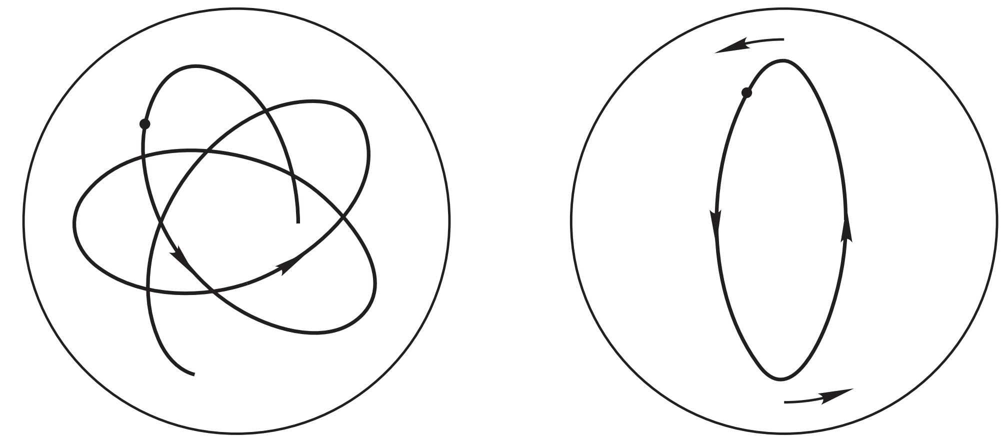
- «Radial-orbit instability» — first time in Merritt & Aguilar (1985)



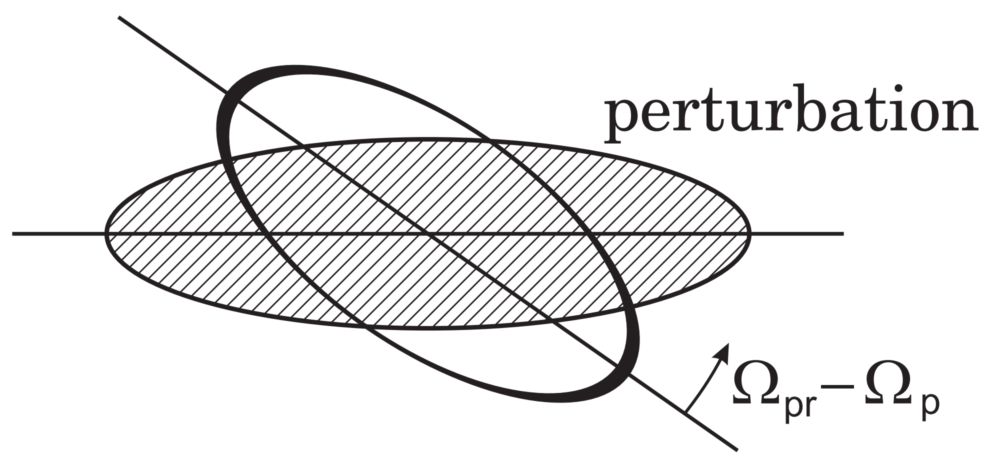
Polyachenko & Shukhman (1972); Merritt (1987)

Orbit species — precession

- A particle trajectory =
  = closed orbit + precession



- Prograde orbits, $\Omega_{pr} > 0$
- Retrograde orbits, $\Omega_{pr} < 0$
- Donkey orbits, $d\Omega_{pr} / dL < 0$



- **Necessary condition for ROI:** $\Omega_{pr} > 0$, $d\Omega_{pr} / dL > 0$

Lynden-Bell (1979), Polyachenko (1991)

Full and ‘slow’ matrix eqs.

- Full matrix equations — double sum:

$$0 = \mathrm{Det}\left\|\delta_{\alpha\beta} - M_{\alpha\beta}(\omega)\right\|$$

$$M^{\alpha\beta}(\omega) = -4\pi\, G\,(2\pi)^2 \sum_{l_1=-\infty}^{\infty} \sum_{l_2=-l}^{l} D_l^{l_2} \times$$

$$\times \int\int \frac{dE\,dL}{\Omega_1} F(E,L) \left[(l_1\Omega_1 + l_2\Omega_2)\frac{\partial}{\partial E} + l_2 \frac{\partial}{\partial L}\right] \left(\frac{L\,\psi^{\alpha\beta}_{l_1\,l_2}}{\omega - l_1\Omega_1 - l_2\Omega_2}\right)$$

- Orbital approach (e.g., Palmer 1994):
  - DF peaking at low L; for nearly radial orbits $\Omega_{pr} = \Omega_2 - \Omega_1/2 \ll \Omega_1$
  - assume ‘slow’ instability, $\omega \sim \Omega_{pr}$: «slow» matrix eq., $2l_1 = l_2$
- No reason for $\omega \sim \Omega_{pr} = \Omega_2 - \Omega_1/2 \ll \Omega_1$. Solve FULL eq.
- Need for a small parameter — dominating harmonic or Kepler potential

$$\epsilon = \frac{M}{M_*}$$ M_* — BH mass or enclosed halo mass, $$M_* = \frac{R^3\Omega_0^2}{G}$$

P & Shukhman (2015)

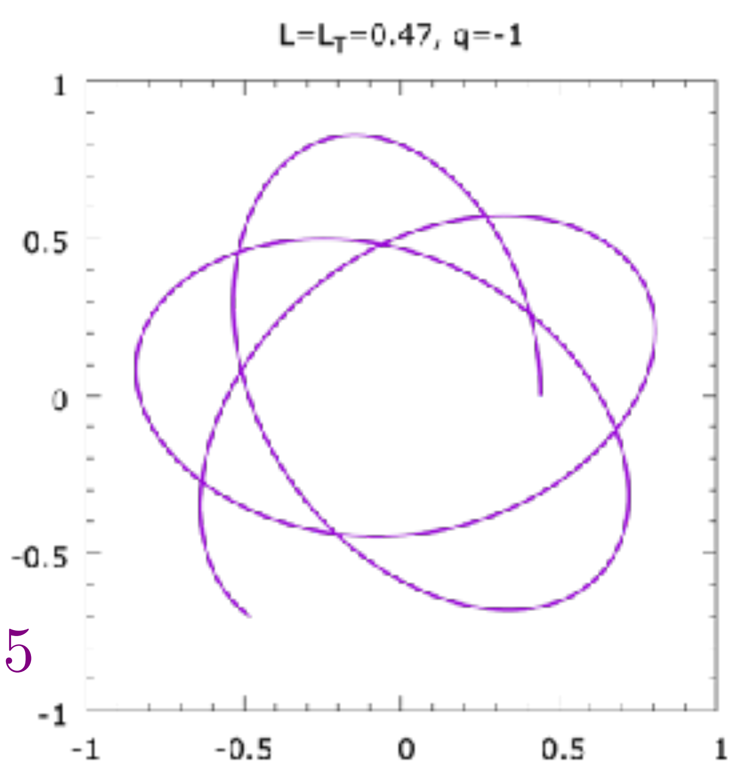
How eccentric «radial orbits» could be?

Softened polytropes

$$f = CH(L_T - L)(-E)^q$$

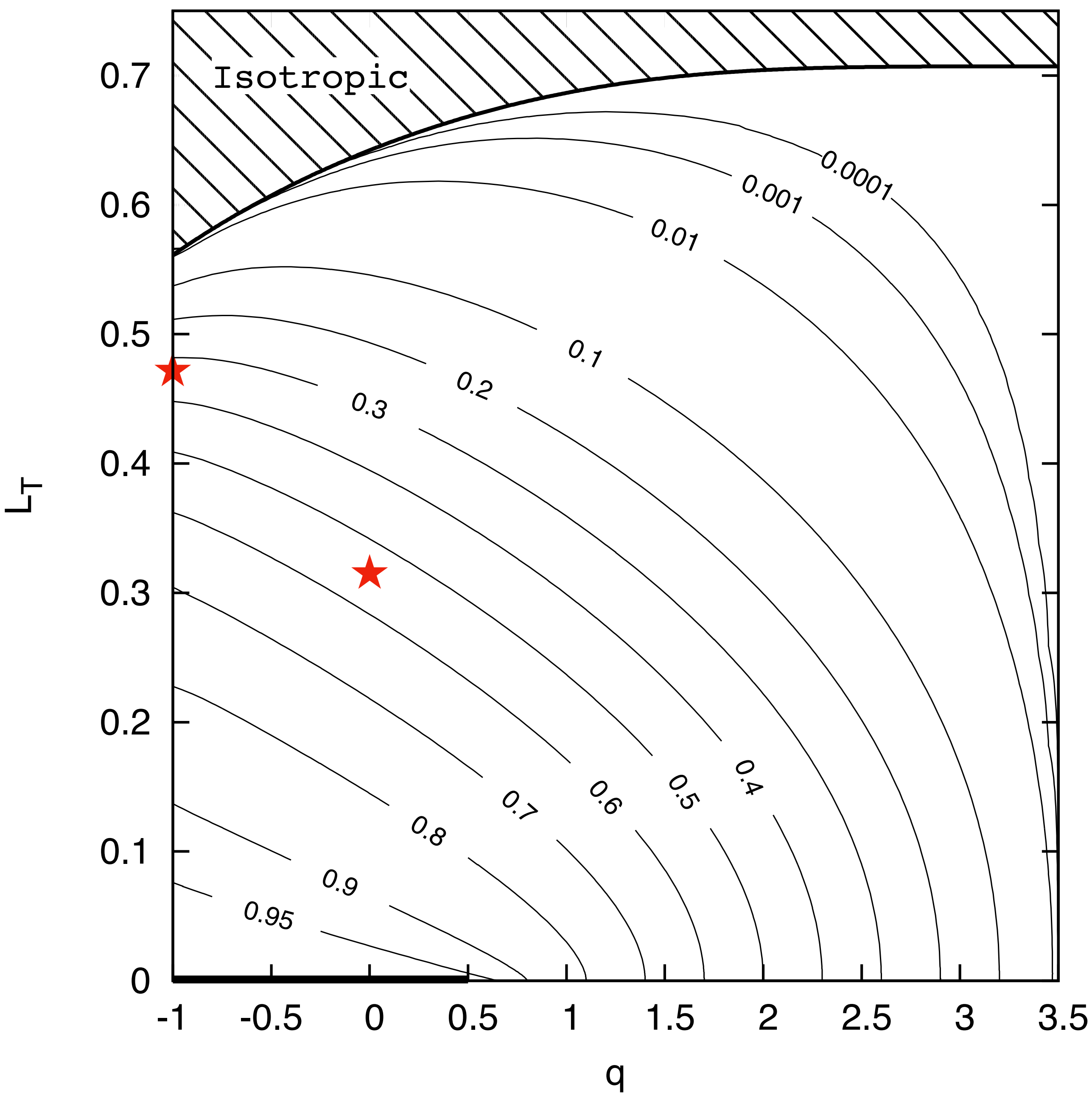
$q = 0, \quad L_{\mathrm{T}} < L_{\mathrm{crit}} = 0.316, \zeta = 2.2$

$q = -1, L_{\mathrm{T}} < L_{\mathrm{crit}} = 0.47, \quad \zeta = 1.5$



$$1 - \frac{r_{\mathrm{min}}}{r_{\mathrm{max}}} \approx 0.5$$

$\beta \equiv 1 - 1/\zeta$



P & Shukhman (2015)

Is orbital picture adequate in the pure radial limit?

We need

- Slow orbital precession:

  $\mathrm{Re}\,\omega \sim \Omega_{pr} = \Omega_2 - \Omega_1/2 \ll \Omega_1$

- Slowly growing perturbation:

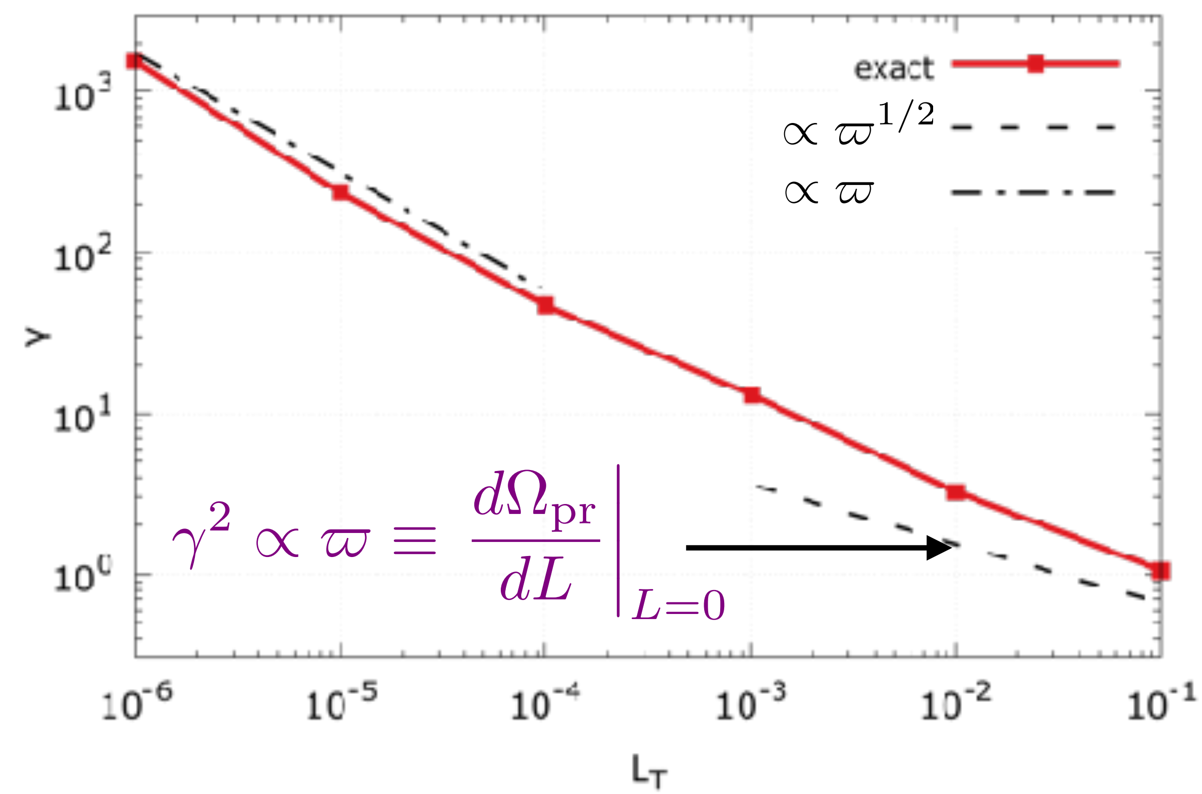
  $\gamma = \mathrm{Im}\,\omega \ll \Omega_1 = 2.16$ (for q=-1)

The same for q=-1/2.
Orbital approach is invalid!

We have

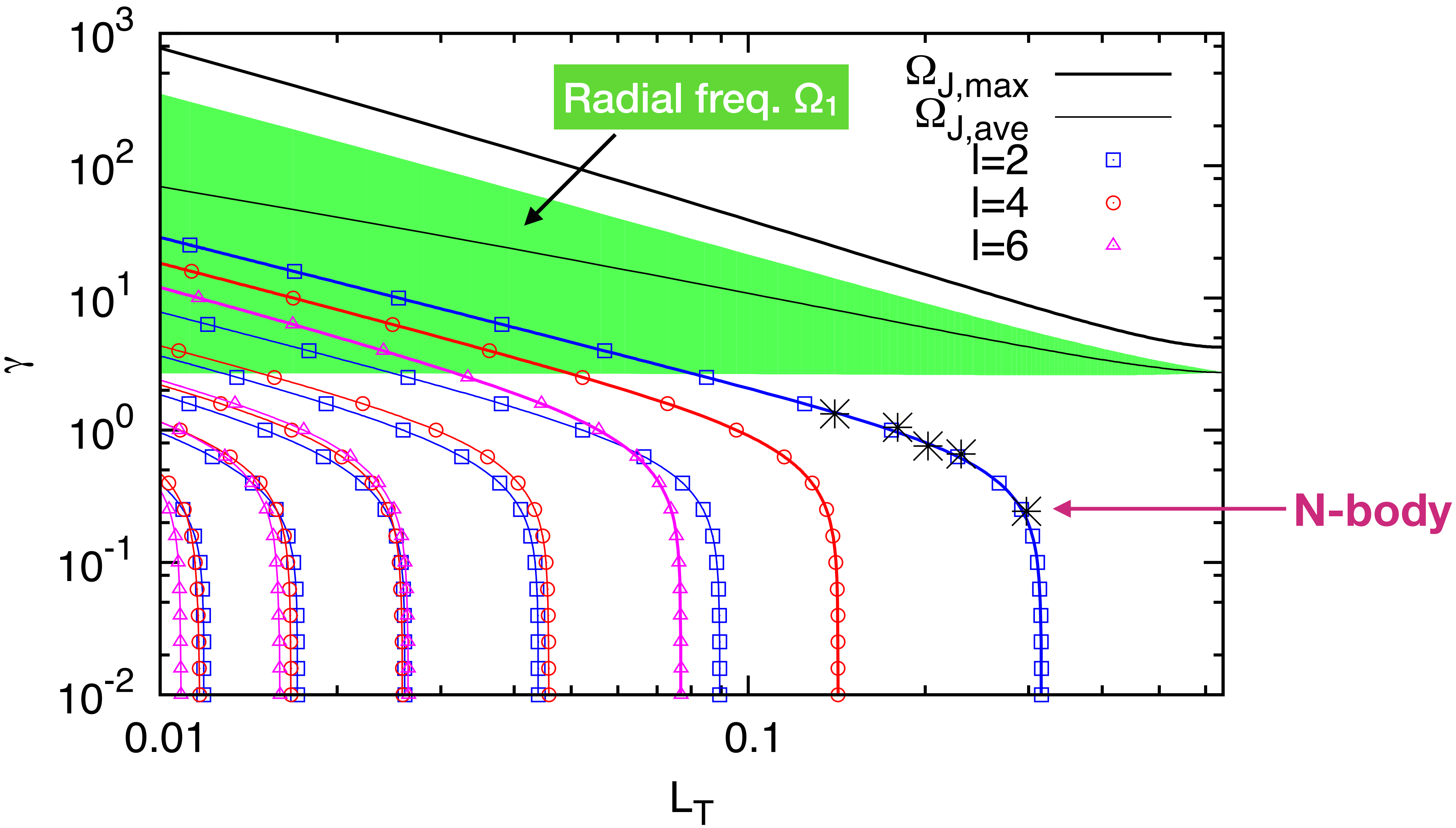
- Enormous growth rates
- Odd spherical harmonics, l = 1,3,5
- Orbital approach is valid for $L_T \geq 0.1$

$q = -1, L_T \to 0$



P & Shukhman (2017); Polyachenko (1991)

One more example (q=0)



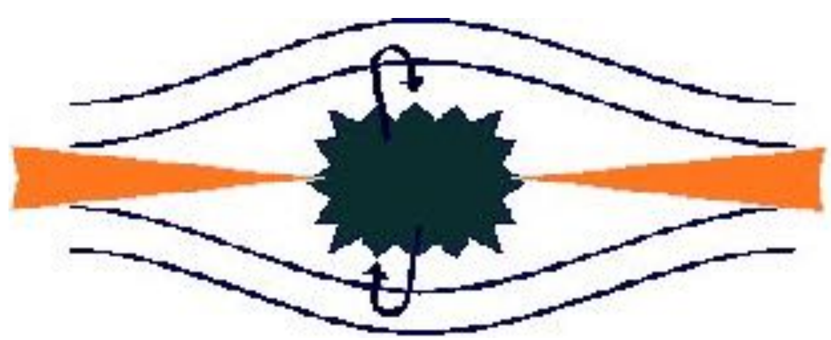
- Maximum growth rates $\gamma = \mathrm{Im}\,\omega \ll \Omega_1$
- l even only

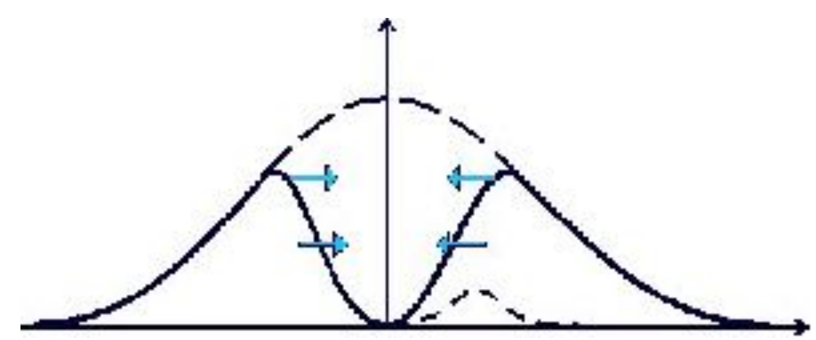
Orbital approach is valid

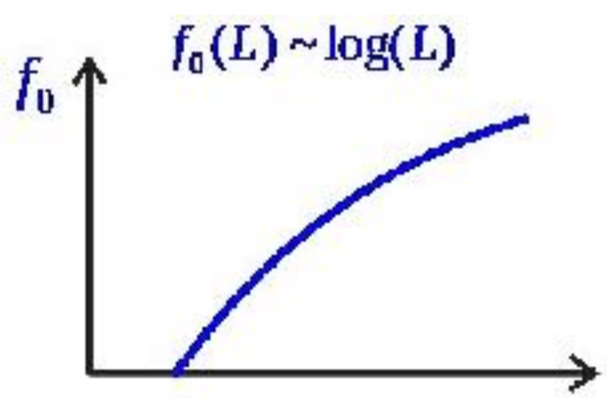
P & Shukhman (2015)

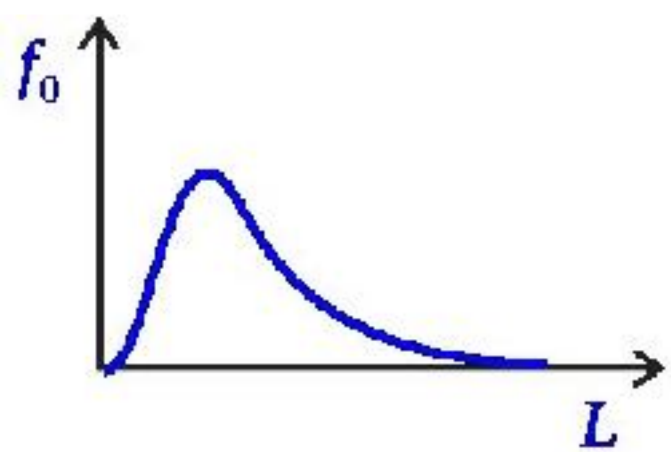
Near-Keplerian systems — stable?

- Retrograde precession. Always for spheres! ∴ stable to ROI
- Another instability? Check **LOSS CONE instability (LCI)**
- Retrograde precession **is welcome and required**









Tremaine (2005), P+ (2007, 2010)

Is LCI near a SMBH possible?

- **No** — for dipole and quadrupole harmonics
- **Yes** — for octupole and higher harmonics $(l \geq 3)$
- L-distribution should be **non-monotonic**
- Stability criterion

Tremaine (2005), P+ (2007, 2008, 2010)

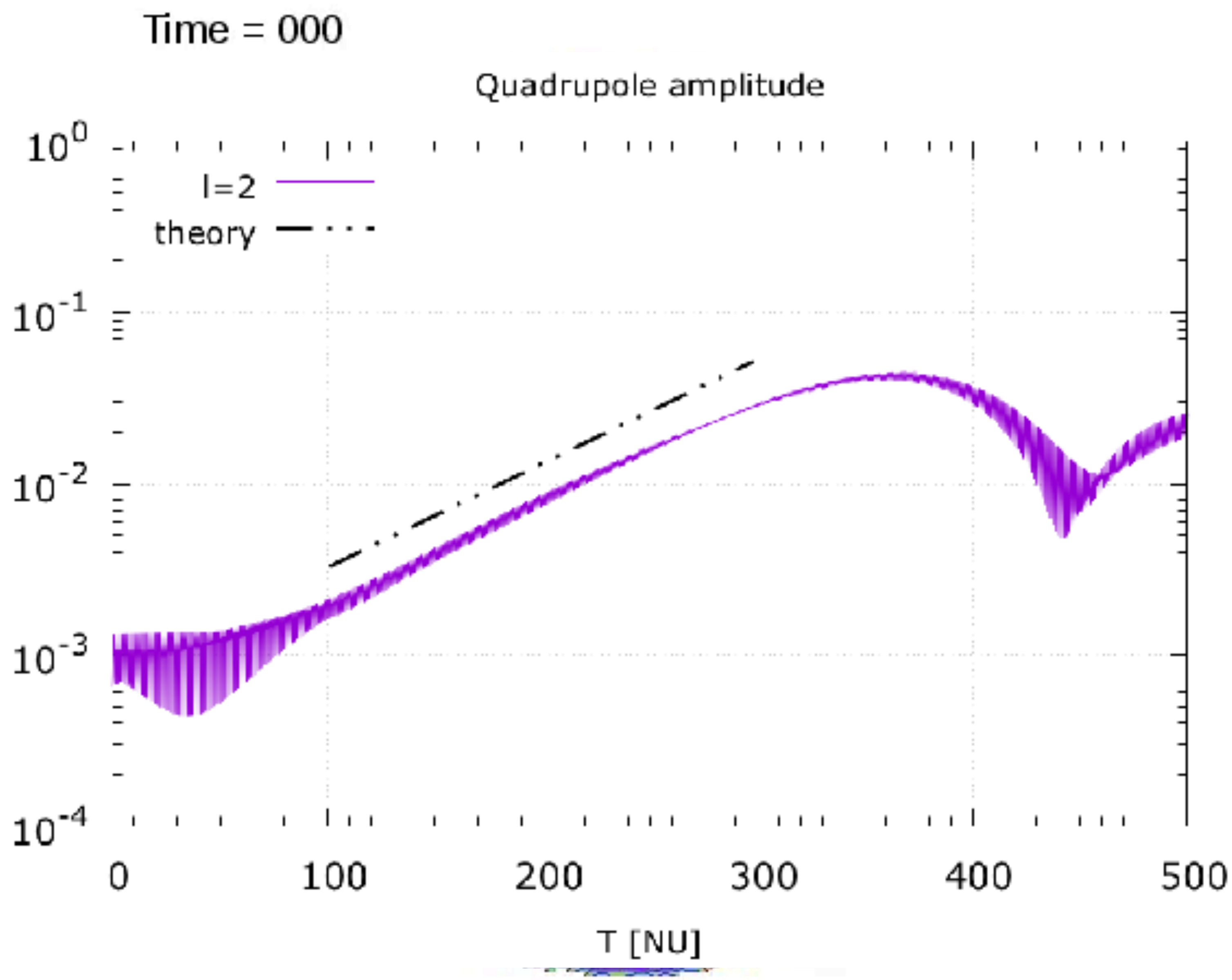
LCI in massive haloes — theory

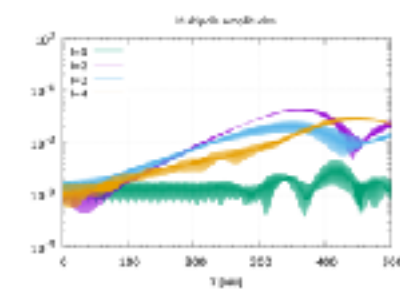
$F(E, L) = A\delta(E - E_0)f(\alpha)$ $\qquad \alpha \equiv L/L_{\rm circ}$ $\qquad f(\alpha) \propto \alpha^n$

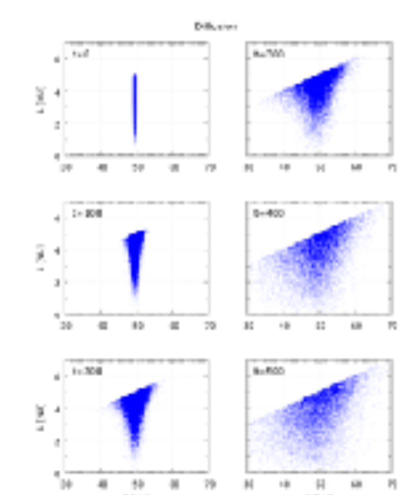
tangentially anisotropic

P+ (2010)

LCI in massive haloes — N-body





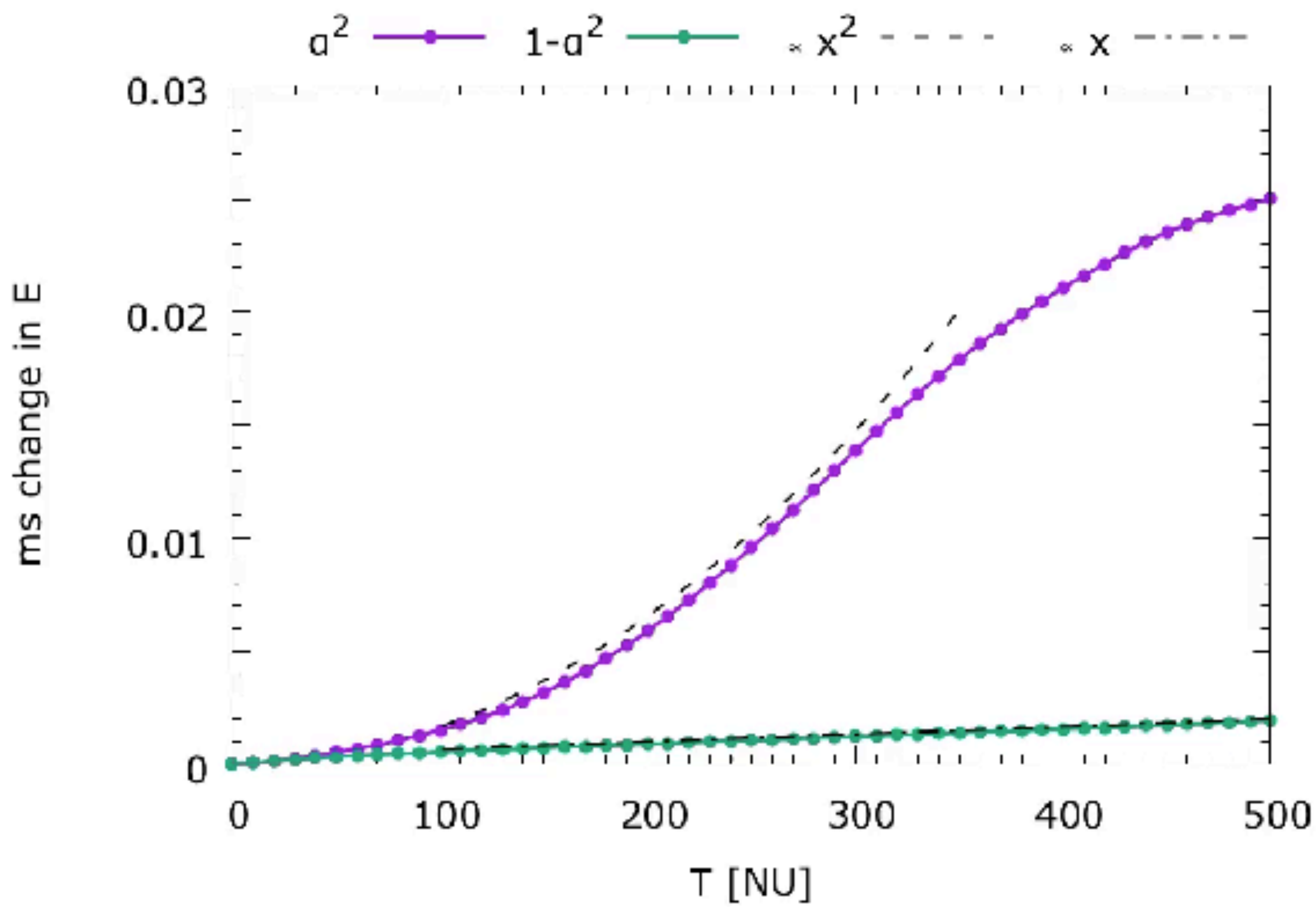


P+ (2017)

Anomalous diffusion — N-body

- Mean square E-change for unstable $f = \alpha^2$ v.s. stable $f = 1-\alpha^2$:

$$W_{\mathrm{C}}^{-2} \sum_i E_i^2 \qquad W_{\mathrm{C}} \equiv \sum_i m_i \mathbf{x}_i \cdot \mathbf{a}_i$$



P+ (2017)

Summary

- Orbital paradigm is restrictive. Jeans instability in anisotropic systems is more general.
- No simple stability criterion. Eigenmode calculations require the full matrix equation, unless a small parameter appears in the problem.
- Tangentially-anisotropic models can be unstable (LCI).
- LCI in near-K potentials may give rise to higher harmonics (starting from octupole). Non-monotonic in L distributions required, but not sufficient.
- LCI affects the diffusion rate rather that the shape.

Thanks for watching !