Progress of high performance simulations of an accretion disk surrounding a supermassive black hole

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General project

STARDISK

Simulations of an SMBH in an AGN with the surrounding star-cluster(Nbody, see [Just et al., 2012, Kennedy et al., 2016])
Accretion disk of black hole uses modified Shakura-Sunyaev model (No Simulation)
Result: Accretion rate increased by presence of Accretion disk

My contribution

Hydrodynamical simulation of the disk instead of just enforcing model.
Interactions of Disk with Star-cluster, mutual feedback
Calculation of star-crossings(heating expected)

STARDISK project

Figure: Artist's impression of a super massive black hole with an accretion disk, Image credit: NASA/JPL-Caltech

STARDISK project

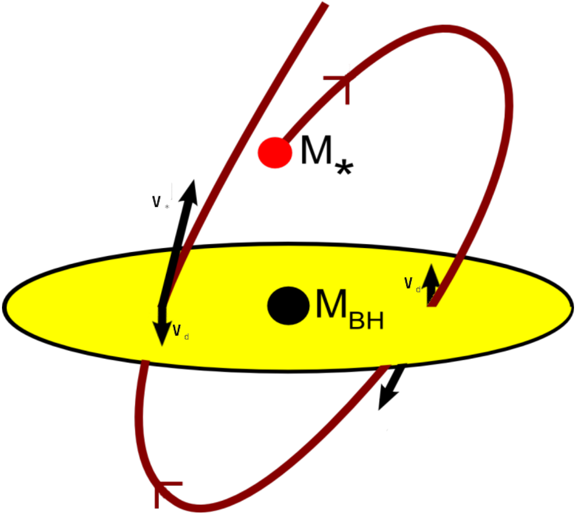

Figure: Figure illustrating the STARDISK situation, Drawing by Gareth F. Kennedy, modifications by Bekdaulet Shukirgaliyev

Status of STARDISK, focused on Disk-Model

Paper I

[Just et al., 2012]

- Stationary Keplerian rotating disk
- Disk has constant thickness
- Star accretion rate enhanced by STARDISK interaction compared to pure stellar dynamics

Paper II

[Kennedy et al., 2016]

- Linear thickness in R compared to constant thickness
- Detailed orbit analysis

α disks

Foundations

[Novikov and Thorne, 1973, Shakura and Sunyaev, 1973]

- Assume turbulent disk with turbulent velocity v_T
- model effect of magnetic field H by viscosity tensor σ
- Amount of turbulence(compare to speed of sound c_S^2) determines angular momentum transport by viscosity

$$\alpha = \frac{v_T}{c_S} + \frac{H^2}{4\pi\rho c_S^2} \tag{1}$$

Details

- Generally $\alpha < 1$
- Different regions by p_{rad} in comparison to p_{gas}
- Process of opacity Thompson scattering, free-free
- Only $\sigma_{R\varphi} = \alpha\rho c_S$ relevant

α disks

Features

- Predicts radial and to some degree z profiles for density, temperature etc.
- Is able to partly explain AGN spectra
- Is theoretically implied to work for non-self-gravitating disks with local MHD turbulences [Balbus and Papaloizou, 1999]
- Radiation transport, radiation pressure, viscosity and hydrodynamics necessary
- Accretion flow and rate determined by α parameter

Governing equations

- Need: modified Navier-stokes equations of hydrodynamics
- Ideal equation of state is used to close the system
  $p = \rho\sigma_{\mathrm{rms}}^2 = \rho\frac{k_{\mathrm{B}}}{\mu_{\mathrm{mol}} m_{\mathrm{H}}} T$, also used to initialise temperature
- Radiation transport included in Flux-Limited-diffusion limit
- Currently: Region such that self-gravity can be neglected

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \tag{1}$$

$$\frac{\partial\mathbf{m}}{\partial t} + \nabla(\mathbf{m}\cdot\mathbf{v}) + \nabla p - \nabla\sigma = \rho\mathbf{g} + \rho\frac{\kappa}{c}\mathbf{F} \tag{2}$$

$$\frac{\partial E}{\partial t} + \nabla(E\mathbf{v}) + \nabla(p\mathbf{v}) - \nabla\cdot(\sigma\mathbf{v}) = \mathbf{m}\cdot\mathbf{g} + \rho\frac{\kappa}{c}\mathbf{v}\cdot\mathbf{F} + \rho c\kappa(aT^4 - E_{\mathrm{R}}) \tag{3}$$

$$\frac{\partial E_{\mathrm{R}}}{\partial t} + \nabla\cdot(E_{\mathrm{R}}\mathbf{v}) = -\nabla\cdot\mathbf{F} - \rho c\kappa(aT^4 - E_{\mathrm{R}}) \tag{4}$$

Equilibrium conditions derived from force Equilibrium [Lodato, 2008]. $v_r = v_\vartheta = 0.0$.

Used Code

- Employ finite volume grid based code PLUTO [Mignone et al., 2007]
- Use self-gravity and radiation transport modules as well more PLUTO additions developed by Rolf Kuiper [Kuiper et al., 2010]
- Make use of heavy MPI parallelisation provided by PLUTO and the modules

Grid and Units

Units

Chosse M31 as given in [Kennedy et al., 2016]

$$M_{\mathrm{BH}} = 1.5 \times 10^{8}\,\mathrm{M}_{\odot}$$
$$M_{\mathrm{Disk}} = 1.5 \times 10^{7}\,\mathrm{M}_{\odot}$$

Grid

- Spherical coordinates r, θ, φ
- Logarithmic grid in r, uniform in θ and φ

$r \in [0.11\,\mathrm{pc}, 2.95\,\mathrm{pc}]$, $\theta \in [0.472\,\pi, 0.527\,\pi]$, $\varphi \in [0, 2\pi]$

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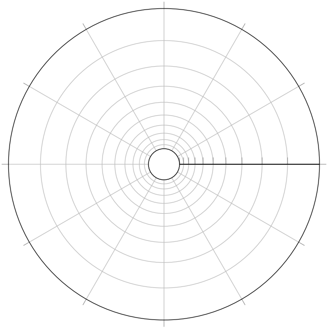


Figure: View from above on r, φ grid

Parameter collection

- $R_{\mathrm{SW}} = \frac{2GM_{\mathrm{BH}}}{c^2} \approx 3.589 \times 10^{-5}\,\mathrm{pc}$ Schwartzschild radius
- $M_{\mathrm{Cl}} = 1.5 \times 10^9\,\mathrm{M}_\odot$ Nuclear Star Cluster Mass
- $M_{\mathrm{BH}} = 1.5 \times 10^8\,\mathrm{M}_\odot$ Black Hole Mass
- $M_{\mathrm{d}} = 1.5 \times 10^7\,\mathrm{M}_\odot$ Total Disk Mass
- $R_{\mathrm{d}} = 25\,\mathrm{pc}$ Outermost radius of the disk
- $R_{\mathrm{sg}} = \left(\frac{1}{2-p}\frac{M_{\mathrm{BH}}}{M_{\mathrm{d}}} h_z R_{\mathrm{d}}^{2-p}\right)^{\frac{1}{3-p}} \approx 2.95\,\mathrm{pc}$ Self-gravity radius, starting from which self-gravity is important
- $h_z = 1.0 \times 10^{-3} R_{\mathrm{d}} = 0.025\,\mathrm{pc}$ Scale heigth
- $h(R) = \begin{cases} \frac{h_z}{R_{\mathrm{sg}}} R \text{ if } R < R_{\mathrm{sg}} \\ h_z \text{ else} \end{cases}$ Linearly varying scale height in BH gravity dominated area

$R = r \sin\theta$ cylindrical radius, $T_{\mathrm{K,\ inner}} = 2.93 \times 10^2\,\mathrm{yrs}$, $T_{\mathrm{K,\ outer}} = 3.88 \times 10^4\,\mathrm{yrs}$

Current initial conditions

First Goal: Equilibrium initial conditions $\Rightarrow$ Stationary disk
Derive stationary state from system of equations in 3 spatial dimensions [Lodato, 2008, Just et al., 2012, Kennedy et al., 2016].

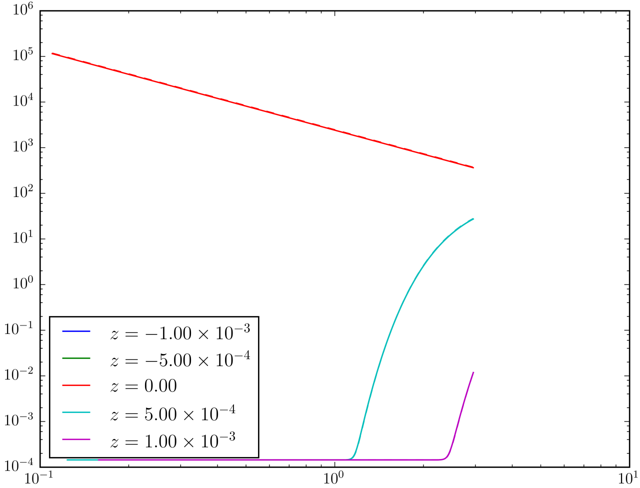
$$\rho(r,\theta) = \rho_{R_{0\rho}} \left(\frac{r}{R_{0\rho}}\right)^{\beta_\rho} \exp\left(-\frac{\cos^2\theta}{2a^2_{R_{0\mathrm{as}}}}\left(\frac{r}{R_{0\mathrm{as}}}\right)^{-2y_{\mathrm{as}}}\right)$$

$$v_\varphi = r\sin\theta\sqrt{\frac{GM_{\mathrm{BH}}}{r^3}}\sqrt{\left(1+\frac{\beta_\rho+\beta_T}{\gamma_{\mathrm{gas}}}\frac{h^2}{r^2\sin^2\theta}\right)}$$

$$c_S = \frac{h}{r\sin\theta}v_\varphi \quad p = \frac{1}{\gamma_{\mathrm{gas}}}c_S^2\rho$$

We always choose $y_{\mathrm{as}} = 0$, $a = \frac{h}{R}$. Also, I infer $\rho_{\mathrm{min}} = 1.0\times10^{-23}\,\mathrm{g\,cm^{-3}}(6\mathrm{H\ cm^{-3}})$, Currently freezing in region with lower density initially.

Current initial conditions



Boundary conditions

r

- Beginning: Outflow no inflow, End Outflow no inflow
- Zero gradient in v_θ
- v_φ: Modified Keplerian gradient towards computational domain

θ

- Beginning: Outflow no inflow, End Outflow no inflow
- Zero gradient in v_r
- v_φ: Modified Keplerian gradient towards computational domain

φ

Periodic for end and beginning

Boundary conditions

Modified Keplerian gradient towards computational domain(example r):
Loop over all cells of the boundary zone, 2 cells in r extended over entire θ and φ.

$$v'_\varphi = v_\varphi \sqrt{\frac{r_{\text{outermost}}}{r_{\text{current}}}} \tag{5}$$

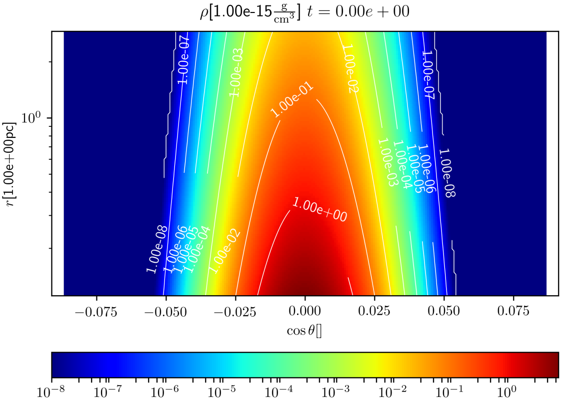
Limit speed to modified Keplerian speed

$$v''_\varphi = \text{Min}(v'_\varphi, v_{\text{K, modified}}) \tag{6}$$

2D test without radiation pressure and self-gravity deactivated

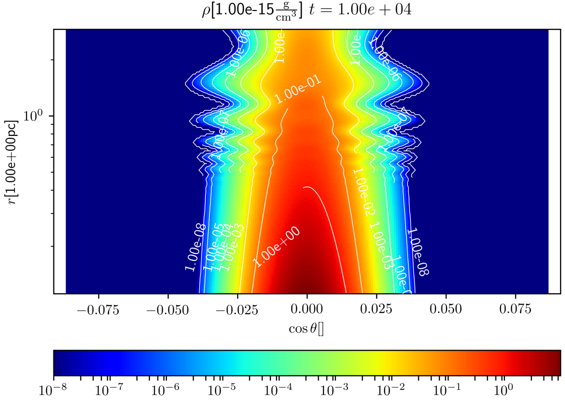
Start with 2D test, $\alpha = 5.0 \times 10^{-2}$, everything apart from self-gravity activated.
Runs fine up to at least 5×10^5 yrs

2D test without radiation pressure and self-gravity deactivated



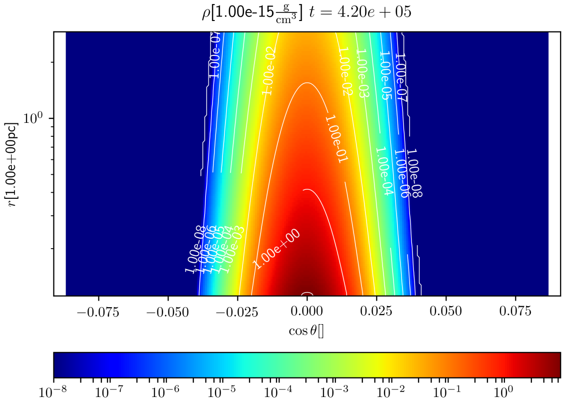
Initial conditons

2D test without radiation pressure and self-gravity deactivated



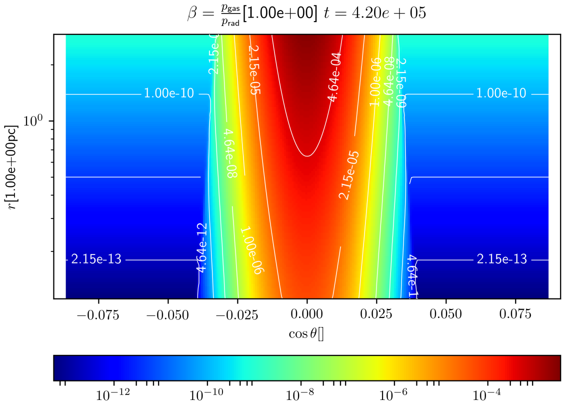
Initial oscillations

2D test without radiation pressure and self-gravity deactivated



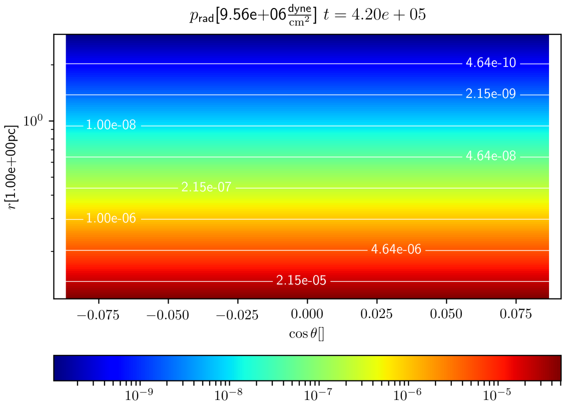
Equilibrium

2D test without radiation pressure and self-gravity deactivated



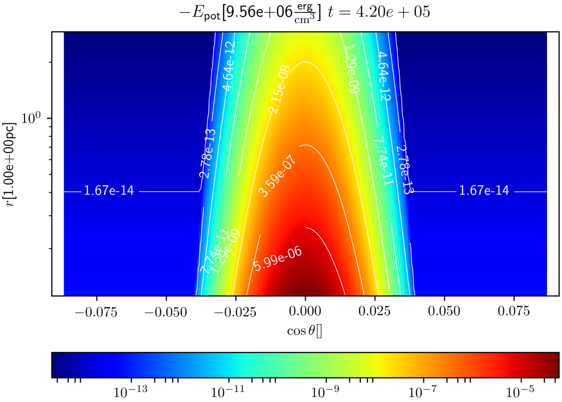
Radiation pressure dominates

2D test without radiation pressure and self-gravity deactivated



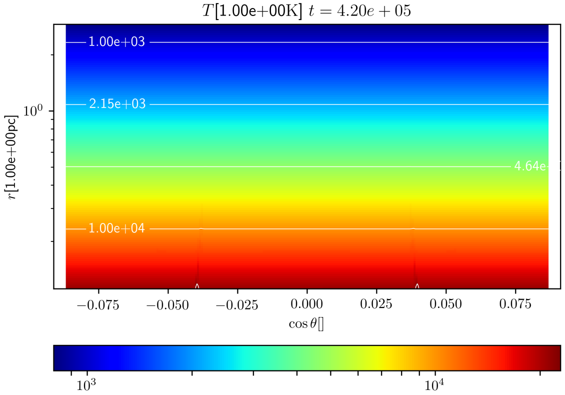
Radiation pressure dominates

2D test without radiation pressure and self-gravity deactivated



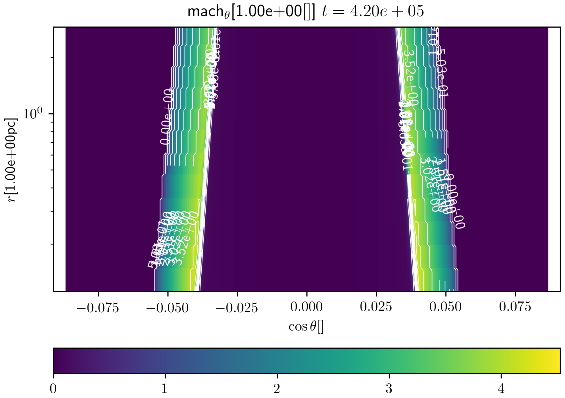
Greater than potential energy

2D test without radiation pressure and self-gravity deactivated



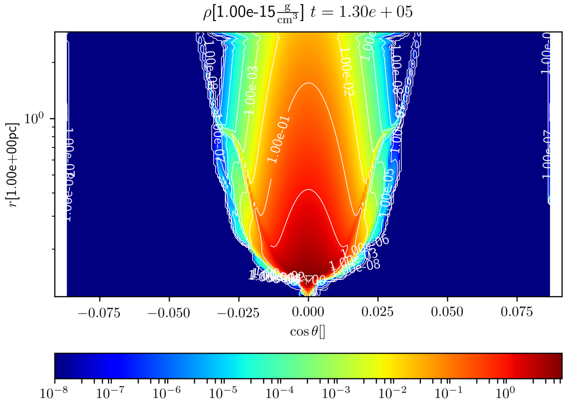
Temperature unchanged

2D test without radiation pressure and self-gravity deactivated



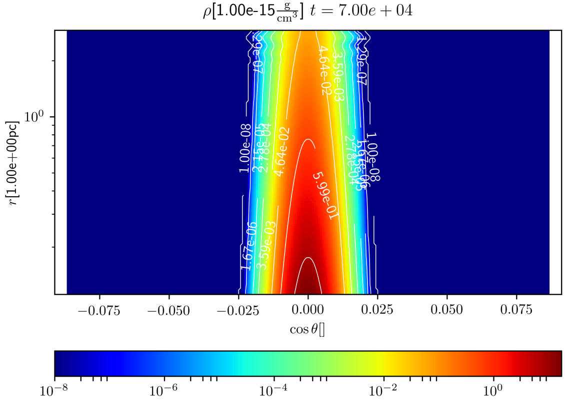
Shock in low density area

Problem: Too high radiation pressure causes explosion



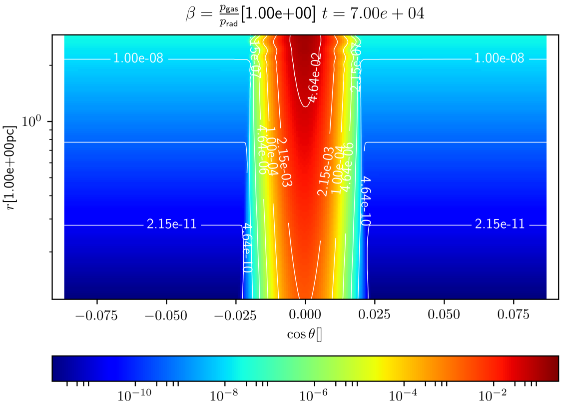
Explosion from much higher pressure in system

Idea: Reduce initial pressure by factor of 3



Results in thinner disk

Idea: Reduce initial pressure by factor of 3



Somewhat better, but still direct crash

Determination of critical temperature

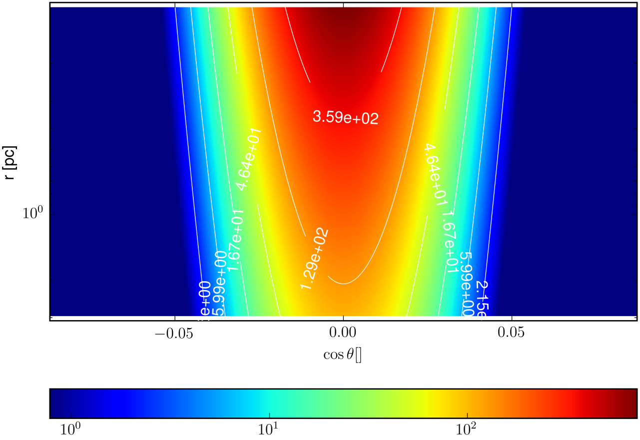
Assume density profile and ideal eos, calculate Temperature

$$p = \rho \frac{k_B T}{\mu m_H} \tag{7}$$

$$\beta = \frac{p_{gas}}{p_{rad}} = \rho \frac{k_B T}{\mu m_H} \frac{3}{a_{rad} T^4} \tag{8}$$

$$T_{crit} = \sqrt[3]{\frac{1}{\beta} \rho \frac{3 k_B}{\mu m_H a_{rad}}} \tag{9}$$

Determination of critical temperature



Radiation pressure equilibrium

$$\frac{\partial p_{\text{rad}}}{\partial r} = -\rho \frac{GM}{r^2} \tag{7}$$

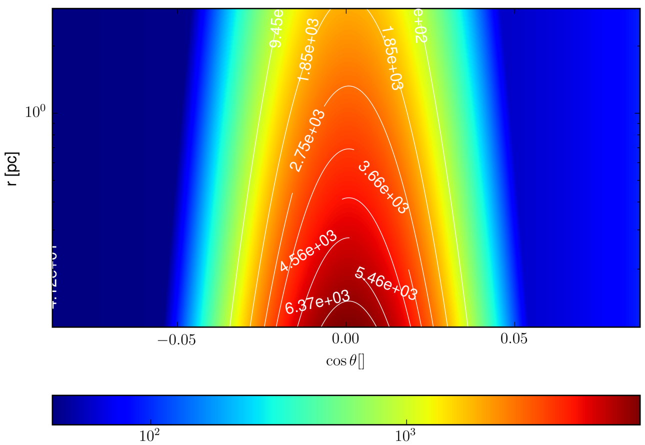
$$4a_{\text{rad}} T^3 \frac{\partial T}{\partial r} = -\rho \frac{GM}{r^2} \tag{8}$$

$$T(r) = \sqrt[4]{\frac{a_{\text{rad}} c_1 r + \rho GM}{a_{\text{rad}} r}} \tag{9}$$

$$c_1 = \frac{a_{\text{rad}} T_0^4 r_0 - GM\rho(r_0, \theta)}{a_{\text{rad}} r_0} \tag{10}$$

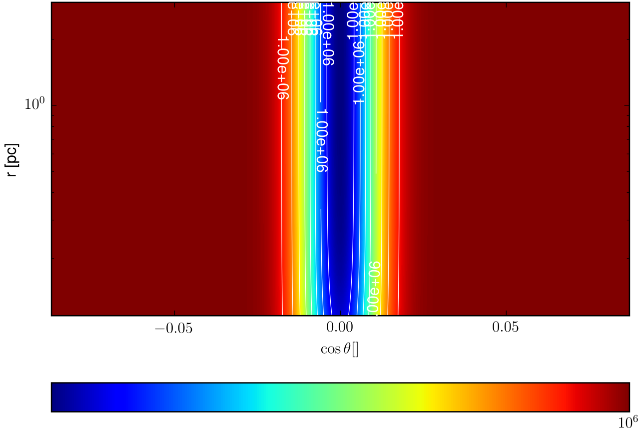
Assume $c_1 = 0.0$, Trying to get it via boundary, some negative values

Radiation pressure equilibrium



with $c_1 = 0$

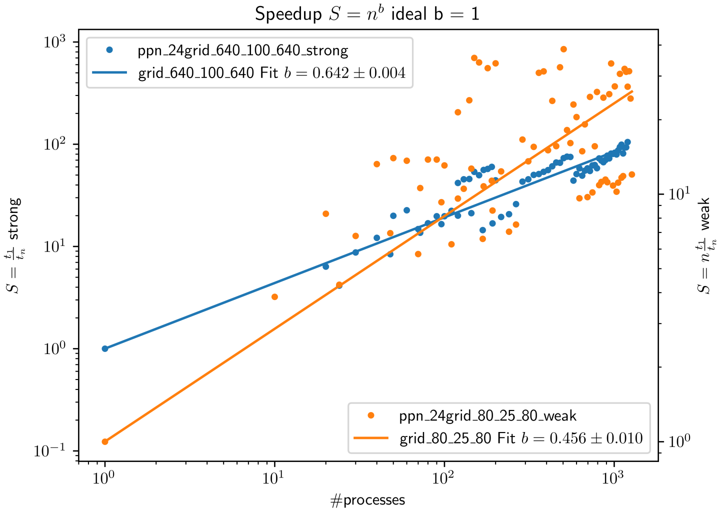
Radiation pressure equilibrium



with c_1 chosen such that $T(0.11\,\mathrm{pc}) = 1.0 \times 10^6\,\mathrm{K}$

Current situation/problems

- Disk goes to equilibrium, stable for long periods of time
- Radiation pressure very high, theoretically cannot be ignored.
- Causes explosion, no equilibrium any more
- Self-Gravity needs to be tested



Strong scaling OK, problem with weak scaling

Further goals

- Slowly progress to 3D simulation
- Do some benchmarks
- Extend to region where self-gravity is important
- Introduce heating due to star-crossings and interactions
- Determine spectra from radiation transport results
- Far goal: Include Hydrodynamical simulation in N-Body simulation

Thank you for your attention!

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